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# LETTER TO THE EDITORS

## COMMENTS ON THE PAPER "THE SIMILARITY HYPOTHESIS APPLIED TO TURBULENT FLOW IN AN ANNULUS" BY H. BARROW, Y. LEE AND A. ROBERTS

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IN THEIR RECENT PAPER [1], the method by which Barrow, Lee and Roberts attempt to apply the similarity hypothesis seems to this writer to be unsound in concept.

They propose that the application in turn of their equation (1) to the inner and outer regions of the annulus should result in expressions for the velocity defect in these regions which are independent of the annular radius ratio. So that if the abscissa of the graph on which either of these expressions is plotted has a given value then the velocity has only one corresponding value for all radius ratios.

If this were so, it would be reasonable to expect the expressions to describe correctly the profile in an annulus whose radius ratio was as arbitrarily close to unity as desirable; that is an annulus which was effectively a parallel-plate passage, in which, of course, the inner- and outer-region velocity profiles would be identical, as must be the velocity defect expressions.

Thus it seems to this writer that the postulate of radius-ratio independence is contrary to the postulate of different velocity defect expressions for the inner and outer regions [Figs. 2(a) and 2(b)].

Further, the authors argue at some length that the poor agreement of their equation (11) with their experiments in the inner region is due to the assumption concerning the turbulence characteristics not being valid in the inner region. It is noteworthy that neither does it agree with the experimental results quoted by Goldstein ([2], p. 352) for channel flow where this assumption is at least as valid as it is in the outer region. According to the authors' postulate of radius-ratio independence of their velocity defect expressions, their equation (11) should agree if the assumption concerning turbulence characteristics is valid.

It is perhaps pertinent to this observation to note that the authors' equation 1 (ii) is stated by them to be an expression for the shear stress, whilst equation 1 (iii) is used to derive to velocity defect expressions.

Goldstein ([2], p. 354) states explicitly that equations 1 (ii) and (iii) are to be regarded as equal possibilities from which velocity expressions can be derived; the first leading to an equation of motion similar to Prandtl's momentum theory, and the second, to an equation similar to that from Taylor's vorticity transfer theory with symmetrical turbulence. The second, used by the authors, is shown to give poor agreement with channel flow.

Finally it could be argued that the authors' approach is justified in a heuristic manner by considering the agreement of their theory and experiments in Fig. 2(a). However the experimental results of Brighton and Jones [3] in the outer region show a notable effect of radius ratio and a very pronounced effect of Reynolds number.

### REFERENCES

1. H. BARROW, Y. LEE and A. ROBERTS, *Int. J. Heat Mass Transfer* **8**, 1499–1505 (1965).
2. S. GOLDSTEIN (editor), *Modern Developments in Fluid Mechanics*. O.U.P., London (1938).
3. J. A. BRIGHTON and J. B. JONES, *J. Bas. Engng* **86**, 835 (1964).

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## AUTHORS' REPLY

WE ARE pleased to have Mr. A. Quarmby's comments on our paper, and to learn that he has given careful consideration to its contents.

In our paper, we have derived expressions for the velocity defects in the inner and outer regions of an axisymmetric turbulent annular flow, employing Goldstein's [1] expressions for $l$ and $M$ (viz. equation 1 (i) and 1 (iii) in the paper). As stated, equation 1 (iii) was chosen in preference to 1 (ii), which is an expression for the shearing stress, because in the case of axisymmetric pipe flow it leads to a better result ([1], p. 494). This appeared to be a logical choice for the study of the annular flow in view of the fact that the pipe geometry can be considered as a particular case of the annular geometry. At least, the flow in both geometries is axisymmetric. At the outset, a postulate concerning the independence of the radius ratio is not made. Indeed, no such condition is